

GEODESIC CORRESPONDENCE BETWEEN RIEMANNIAN SPACES

BY

MORRIS TELES

B.S., Tulane University, 1933

S.M., University of Chicago, 1935

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS
IN THE GRADUATE SCHOOL OF THE UNIVERSITY OF
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The purpose of this paper is the consideration of geodesic correspondences between Riemannian spaces, and the nature of the induced correspondence on corresponding subspaces.

Throughout the discussion there is maintained the hypothesis that the spaces are in geodesic correspondence. It is shown that asymptotic curves (and conjugate directions) are preserved on the subspaces. With the aid of this result the author proves that a necessary and sufficient condition for geodesic correspondence of corresponding hypersurfaces is that corresponding normals be in corresponding directions. From the above a characteristic absolute invariant is deduced for hypersurfaces in geodesic correspondence. These results are generalized for subspaces of fewer dimensions.

It is next shown that the existence of a pair of geodesically corresponding non-totally geodesic hypersurfaces implies the existence of a whole family of geodesically parallel hypersurfaces on which geodesics are preserved. If, further, the pair of hypersurfaces are in isometric correspondence, it is shown that the correspondence is isometric throughout the spaces.

The latter part of the discussion is devoted to a consideration of corresponding curves, their curvatures and osculating k -spaces. By induction it is shown that the latter are invariant under geodesic correspondence of the spaces.

If corresponding curves lie on a pair of geodesically corresponding hypersurfaces immersed in the original spaces it is found that the principal normals of the curves as curves on the hypersurfaces correspond if and only if they correspond on the spaces. The author then derives the differential equation of the system of curves in the spaces whose normals correspond. The nature of this system gives meaning to the following result that if the normals to a space-filling congruence of curves on a pair of corresponding hypersurfaces correspond both on the hypersurfaces and in the enveloping spaces, then the correspondence is geodesic on the hypersurfaces.

There follows an interesting relation between the corresponding curvatures of corresponding curves and the angles between corresponding normals. From this relation is deduced the result that

if the direction corresponding to the p -th normal of a curve in one space is orthogonal to the p -th normal of the corresponding curve in the other space, then the p -th curvatures vanish for both curves.

If the curvatures after the p -th vanish for a given curve the corresponding curvatures vanish for the corresponding curves. If all the normals of a pair of corresponding curves are in corresponding directions the curvatures, after the first, are proportional.

VITA

Morris Teles was born in New Orleans, Louisiana, on December 25, 1912. He received his early education in the public schools of that city, and in 1929 entered Tulane University. He received his B.S. degree in the spring of 1933, and the following fall entered the University of Chicago, where he earned a scholarship for two years. After receiving the master's degree from Chicago he transferred to the University of Illinois, where he taught from 1936-38, meanwhile continuing his work towards the doctorate. He wishes to express his sincerest thanks and appreciation to his adviser, Professor Harry Levy, whose instructive suggestions and criticisms were ever a source of inspiration in the preparation of this dissertation.

